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# Advertising and entry deterrence: how the size of the market matters

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## Abstract

Schmalensee (1974) shows that, when the cost functions of the advertising effort are linear, the monopoly position is only sustained when the advertising's effectiveness of the incumbent is relatively high. We show that this result does not hold in a more general nonlinear cost. The no-entry equilibrium may hold even when the relative effectiveness of the incumbent is low. This happens when the size of the market is sufficiently low.

## 1 Introduction

The central question considered in this paper is how an incumbent responds to the threat of entry by a rival when the advertising variable is the weapon used either by the latter to attract the incumbent's customers and by the incumbent to resist any intrusion. We determine the optimum allocations of advertising effort when the incumbent moves first by determining its advertising effort, and then, the rival considers whether to enter or not. If entering is decided, the optimal entrant's advertising effort is determined. We show that, in equilibrium, the optimal behavior of the incumbent may be to deter entry, depending on the relative effectiveness of its advertising effort and also the size of the market.

A number of papers (Monahan (1987), Schmalensee (1976)) derived the optimal advertising efforts when the revenue of each firm is determined by its relative spending in advertising. However, except a brief analysis by Schmalensee (1974), they are only concerned by the equilibrium marketing expenditure of the existing firms. Schmalensee (1974) who question whether the presence of a dynamic effect of advertising may lead to a barrier to entry, shows that the incumbent needs a strong

advantage for the advertising to be of deterrence entry. However, the type of equilibrium does not depend on the size of the market. We show that this result is due to the linearity of the advertising cost function. In a more general nonlinear advertising cost, the size of the market does matter. An incumbent with a relatively small effectiveness of advertising can sustain monopoly when the size of its market is sufficiently low.

We exclude the price as a marketing decision variable. This assumption can be justified in many markets where there is little price competition but other factors related to the marketing variable are significant. In the pharmaceutical industries, brands use advertising rather than price to influence the post-patent competition and then the entry decision of generic firms (Scott Morton (2000)).

The remainder of the paper is organized as follows: In the section 2, we specify the model and notation. Section 3 derives the Stackelberg equilibrium and its properties. Section 4 concludes the paper.

## 2 The model

Consider an incumbent firm  $X$  and a rival  $Y$  who considers the possibility to enter into the incumbent's market. Let  $x$  and  $y$  respectively the advertising spending of the incumbent and the entrant. These quantities measure the number of ads that firms send to consumers<sup>1</sup>. The size of the market  $V$  is fixed. We assume that marginal costs of production are constant and equal for the firms. Let  $m$  denote the difference between the price and marginal cost. The incumbent chooses first its advertising effort  $x$  and then the potential entrant chooses its advertising effort  $y$  under full knowledge of  $x$ . Let  $c_X(\cdot)$  (respectively  $c_Y(\cdot)$ ) be the advertising cost of firm  $X$  (respectively firm  $Y$ ). We assume that  $c_X$  is convex and strictly increasing and similarly for  $c_Y$  with  $c_Y'(0) > 0$ .

In this paper, we assume that the market-share  $S$  of firm  $X$  has the following form:

$$S = \frac{1}{1 + \theta y/x} \quad (1)$$

where  $\theta$  is a positive parameter that indicates the effectiveness of the entrant's advertising against the incumbent's advertising. The market share of firm  $Y$  is  $1 - S$ .

The optimization problem of  $X$  is

$$\text{Maximize} \quad \pi_X = SmV - c_X(x), \quad x \geq 0 \quad (2)$$

and the program of firm  $Y$  is

$$\text{Maximize} \quad \pi_Y = (1 - S)mV - c_Y(y), \quad y \geq 0 \quad (3)$$

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<sup>1</sup>Schmalensee (1976) adopts the same formulation.

### 3 The equilibrium

To analyze the allocation of the advertising spending in this scenario, we begin by considering the second-stage choice of firm  $Y$ . At this second stage, firm  $Y$  takes  $x$  as given and chooses  $y$  solution to the program (3). We are interested in deriving the conditions under which the entry is deterred, that is when  $y = 0$ .

Since  $\pi_Y$  is strictly concave in the strategy  $y$ , the maximizer is unique. The solution is characterized by the Kuhn-Tucker conditions of the maximization problem in (3):

$$\frac{\partial \pi_Y}{\partial y} = -\frac{\partial S}{\partial y}mV - c'_Y(y) \leq 0$$

and

$$y \frac{\partial \pi_Y}{\partial y} = y \left[ -\frac{\partial S}{\partial y}mV - c'_Y(y) \right] = 0$$

These are solved easily, and one obtains:

**Proposition 1** *Let  $x^{\det} = \theta mV / c'_Y(0)$ . Given the advertising spending  $x$  of the incumbent, the optimal reaction  $y$  of the potential entrant is null for  $x \geq x^{\det}$  and strictly positive for  $0 < x < x^{\det}$  with  $x$  and  $y$  verifying*

$$\theta mV x = (x + \theta y)^2 c'_Y(y) \quad (4)$$

In Eq. (4),  $x^{\det}$  is the minimum spending of firm  $X$  to advertising necessary to deter firm  $Y$  from entering in its market. It represents the deterrence advertising effort. Moreover,  $x^{\det}$  depends positively on the size of the market, the gross profit margin and the relative effectiveness of the entrant's advertising. When entry is accommodated and the incumbent choice is fixed, there is a positive relationship between the advertising effort of the entrant and the attractiveness of the market, measured by  $mV$ , but the effect of an increasing of the relative effectiveness of the entrant's advertising is ambiguous.

We consider now the choice of the first stage of the incumbent. At this first stage, the incumbent chooses its advertising spending  $x$  solution to the program (2). When choosing  $x$ , the incumbent firm takes into account the reaction of the potential entrant describing in proposition 1.

If  $x \geq x^{\det}$ , then  $y = 0$  and  $S = 1$ . Eq. (2) implies that the profit of the incumbent is a decreasing function of  $x$ . Consequently, the optimal choice of the incumbent is obtained at  $x$  not greater than  $x^{\det}$ .

Let's begin by showing that the constraint  $x \geq 0$  is not binding. We have:

**Lemma 1** *When  $x \rightarrow 0$  then  $y \rightarrow 0$  and  $y/x \rightarrow +\infty$*

Furthermore, we have:

**Lemma 2** *In the limit as  $x$  approaches zero, the derivative of  $\pi_X$  with respect to  $x$  becomes infinite.*

It follows from lemma 2 that the optimal choice of the incumbent is positive. We show next that the type of equilibrium, with or without entry, depends on the sign of the derivative of  $\pi_X$  at  $x = x^{\det}$ .

Consider first the case in which the derivative  $d\pi_X/dx$  is negative for  $x = x^{\det}$ . The profit function  $\pi_X$  of the incumbent has an interior maximum at a value of  $x$  that satisfies

$$\left( \frac{\partial S}{\partial x} + \frac{dy}{dx} \frac{\partial S}{\partial y} \right) mV = c'_X(x) \quad \text{with } 0 < x < x^{\det} \quad (5)$$

The first term in the left-hand side of Eq. (5) captures the direct effect of greater advertising effort  $x$  by the incumbent on its profit. The second term represents the strategic effect of  $x$  on the choice of the potential entrant. In this case, the incumbent maximizes its profit by choosing its advertising spending such that the marginal revenue equals the marginal cost. Given that this advertising level is smaller than  $x^{\det}$ , the equilibrium is with entry.

Consider second the case in which the derivative  $d\pi_X/dx$  is not negative at  $x = x^{\det}$ . We need the following hypothesis:

**Hypothesis:** The cost function  $c_Y(\cdot)$  is such that, for all  $y > 0$ , either  $c''_Y(y) \leq c''_Y(0)$  or  $c''_Y(y)/c'_Y(y) \leq c''_Y(0)/c'_Y(0)$ .

The cost functions  $c_Y(\cdot) = (1 + y)^a$  with  $a \geq 1$  verify this hypothesis in that  $c''_Y(y)/c'_Y(y) \leq c''_Y(0)/c'_Y(0) \forall y > 0$ .

We have:

**Lemma 3** *Under the above hypothesis, the marginal revenue of the incumbent for all  $x$  such that  $0 < x < x^{\det}$  is superior to its marginal revenue at  $x = x^{\det}$ .*

Let  $R = \frac{\partial S}{\partial x} + \frac{dy}{dx} \frac{\partial S}{\partial y}$ . We deduce from lemma 3 that  $R(x) > R(x^{\det}) \geq c'_X(x^{\det}) \geq c'_X(x)$  for all  $0 < x < x^{\det}$ . It follows that  $d\pi_X/dx$  is positive for  $0 < x < x^{\det}$  and the profit function of the incumbent is then strictly increasing. Consequently, the incumbent chooses an advertising spending equal to  $x^{\det}$  and entry is deterred.

For simplicity, let take  $c_Y'(0) = 1$ . We show easily that the derivative of  $\pi_X$  for  $x = x^{\text{det}}$  is given by

$$\frac{d\pi_X}{dx}(x = x^{\text{det}}) = \frac{1}{\theta(2 + mVc_Y''(0))} - c_X'(\theta mV)$$

**Proposition 2** *When the size of the market  $V$  and gross profit margin are fixed, there exists a threshold level  $\theta^*$  such that the equilibrium is of deterrence entry if  $\theta$  is not greater than  $\theta^*$ . When the relative effectiveness of entrant's advertising and gross profit margin are fixed, there exists a threshold level  $V^*$  such that the equilibrium is of deterrence entry if the size of the market  $V$  is not greater than  $V^*$ . Furthermore, in general<sup>2</sup>,  $\theta^*$  and  $V^*$  are monotonically and negatively related.*

Let fix the effectiveness of firms and vary the profitability of the market measured by  $mV$ . When increasing its advertising spending, the incumbent compares its marginal revenue with its increasing marginal cost. When the profitability of the market is high, the incentive for the incumbent to deter entry is high. But the reaction of the entrant is aggressive for the same reason. The incumbent needs then a high investment in advertising to succeed in protecting its monopoly position. In this case, the marginal cost is superior to the marginal revenue and then the incumbent prefers accommodates entry. In contrast, when the profitability is not high, because of soft reaction of the entrant, the incumbent firm increases its profit by spending more in advertising. In this case, the incumbent prefers a strategy of deterrence entry.

**Corollary 3 (Schmalensee (1974))** *When the marginal costs of the advertising effort are constant, then the type of equilibrium is independent of the size of the market. The equilibrium is of deterrence entry when the relative effectiveness of entrant's advertising is not greater than  $1/2$ .*

**Proof.** We give a second proof that helps to understand what happens special when the advertising cost functions are linear. Let fix the parameter  $\theta$  and consider a change in the size of the market from  $V$  to  $\lambda V$ . Firm X's profit function can be rewritten as  $\pi_X(x, y, V) = S(x, y)mV - c_X(x)$  where  $y$  is the best reaction of the entrant that depends on  $x$  such that the pair  $(x, y)$  verifies Eq. (4). The minimum advertising spending  $x^{\text{det}}$  that dissuades entry is then multiplied by  $\lambda$ . Eq. (4) yields that the entrant must choose an advertising spending equals to  $\lambda y$  when the incumbent chooses the level  $\lambda x$ . Given that

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<sup>2</sup>A sufficient condition is  $c_Y''(y) > 0$ , for all  $y$ .

$\pi_X(\lambda x, \lambda y, \lambda V) = \lambda \pi_X(x, y, V)$ , it follows that the incumbent changes its advertising spending and is optimally multiplied by the same factor  $\lambda$ . Consequently, the type of equilibrium is not modified. ■

## 4 Conclusion

Schmalensee (1974) considers the optimal advertising expenditure of the Stackelberg leader incumbent who may induce the potential entrant not to enter. He shows that the incumbent needs a strong advantage for the advertising's effectiveness to be of deterrence entry. Furthermore, the equilibrium achieved does not depend on the size of the market. In this paper, by considering the advertising measured in units of ads, we allow the advertising cost functions to be nonlinear. We showed that the type of equilibrium depends on the relative effectiveness of advertising, as stated by Schmalensee, and also of the size of the market. In contrast to what stated by Schmalensee, an incumbent firm with a relatively small effectiveness of advertising may resist any intrusion into its market if the size of the market is sufficiently small. Ellison and Ellison (2000) reports evidence that drugs with higher revenues are most likely to attract generic entry. Scott Morton (2000) analyzes the U.S. pharmaceutical industry and question whether pre-expiration brand advertising deters generic entry. She shows that the market attractiveness, measured by pre-expiration brand revenue, is the most important factor determining the number of entrants.

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## A Proof of Lemma 1

Eq. (4) yields  $\theta m V x \geq \theta^2 y^2 c'_Y(y) \geq \theta^2 y^2 c'_Y(0)$ . Then  $0 < y^2 \leq m V x / (\theta c'_Y(0))$ . It follows that  $y \rightarrow 0$  for  $x \rightarrow 0$ . Also, it comes from Eq. (4) that

$\theta y/x = \sqrt{\theta mV/(xc'_Y(y))} - 1$ . Since  $xc'_Y(y) \rightarrow 0$  for  $x \rightarrow 0$ , hence  $y/x \rightarrow +\infty$  for  $x \rightarrow 0$ .

## B Proof of Lemma 2

Let  $R = \frac{\partial S}{\partial x} + \frac{dy}{dx} \frac{\partial S}{\partial y}$ . From equation (4) and by applying the envelope theorem, we obtain the derivative  $dy/dx$  and then we have

$$\begin{aligned} R &= \frac{\theta y}{(x + \theta y)^2} - \frac{c'_Y(y)}{mV} \left\{ \frac{\theta mV - 2(x + \theta y)c'_Y(y)}{2\theta(x + \theta y)c'_Y(y) + (x + \theta y)^2 c''_Y(y)} \right\} \\ &= \frac{\theta y c'_Y(y)}{(x + \theta y)^2 c'_Y(y)} - \frac{c'_Y(y)}{mV} \left\{ \frac{\theta mV - 2(x + \theta y)c'_Y(y)}{2\theta(x + \theta y)c'_Y(y) + (x + \theta y)^2 c''_Y(y)/c'_Y(y)} \right\} \\ &= \frac{y}{x} \frac{c'_Y(y)}{mV} - \frac{c'_Y(y)}{mV} \left\{ \frac{\theta mV(x + \theta y) - 2\theta mVx}{2\theta(\theta mVx) + \theta mVx(x + \theta y)c''_Y(y)/c'_Y(y)} \right\} \\ &= \frac{y}{x} \frac{c'_Y(y)}{mV} - \frac{c'_Y(y)}{mV} \left\{ \frac{\theta mV(\theta y - x)}{2\theta^2 mVx + \theta mVx(x + \theta y)c''_Y(y)/c'_Y(y)} \right\} \\ &= \frac{y}{x} \frac{c'_Y(y)}{mV} - \frac{c'_Y(y)}{mV} \left\{ \frac{\theta mV(\theta y/x - 1)}{2\theta^2 mV + \theta mV(x + \theta y)c''_Y(y)/c'_Y(y)} \right\} \end{aligned}$$

Hence

$$\begin{aligned} R(x) &= \frac{y}{x} \frac{c'_Y(y)}{mV} \left[ 1 - \frac{\theta}{2\theta + (x + \theta y)c''_Y(y)/c'_Y(y)} \right] \\ &\quad + \frac{c'_Y(y)}{mV} \left[ \frac{1}{2\theta + (x + \theta y)c''_Y(y)/c'_Y(y)} \right] \end{aligned} \quad (\text{A})$$

$$\text{Let } A = \frac{y}{x} \frac{c'_Y(y)}{mV} \left[ 1 - \frac{\theta}{2\theta + (x + \theta y)c''_Y(y)/c'_Y(y)} \right] \text{ and } B = \frac{c'_Y(y)}{mV} \left[ \frac{1}{2\theta + (x + \theta y)c''_Y(y)/c'_Y(y)} \right].$$

We deduce from lemma 1 that  $B \rightarrow \frac{c'_Y(0)}{2\theta mV}$  and  $A \rightarrow +\infty$  for  $x \rightarrow 0$ .

## C Proof of Lemma 3

Let retake Eq. (A). For  $x = x^{\text{det}}$ ,  $R = A + B$  with  $A = 0$  and  $B = \frac{c'_Y(0)}{mV} \left[ \frac{1}{2\theta + x^{\text{det}} c''_Y(0)/c'_Y(0)} \right]$ . If  $0 < x < x^{\text{det}}$  we have  $A > 0$ . It suffices to show that

$$\frac{c'_Y(y)}{mV} \left[ \frac{1}{2\theta + (x + \theta y)c''_Y(y)/c'_Y(y)} \right] \geq \frac{c'_Y(0)}{mV} \left[ \frac{1}{2\theta + x^{\text{det}} c''_Y(0)/c'_Y(0)} \right]$$

As  $c'_Y(y) \geq c'_Y(0)$  and the hypothesis holds, we need to show that  $(x + \theta y) \leq x^{\text{det}}$  for each  $(x, y)$  verifying Eq. (4). By Eq. (4) we have  $\theta mVx = (x + \theta y)^2 c'_Y(y)$  and  $\theta mVx^{\text{det}} = (x^{\text{det}})^2 c'_Y(0)$ . For  $0 < x < x^{\text{det}}$ ,  $(x + \theta y)^2 c'_Y(y) \leq \theta mVx^{\text{det}} = (x^{\text{det}})^2 c'_Y(0) \leq (x^{\text{det}})^2 c'_Y(y)$ .